

Dynamic Stability Of Layered Beams With Bolted Joints Under Parametric Excitation

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Bachelor of Technology

In

Mechanical Engineering

By

Shivakumar Satyanarayan & Amarendra Kumar Patel



Department of Mechanical Engineering

National Institute of Technology

Rourkela

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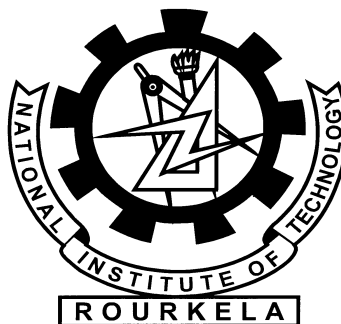
Mechanical Engineering

By

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Rourkela**

CERTIFICATE

This is to certify that the thesis entitled “**Dynamic Stability of Layered Beams With Bolted Joints Under Parametric Excitation.**” submitted by **Shivakumar Satyanarayan, Roll No: 10303013** and **Amarendra Kumar Patel, Roll No: 10303038** in the partial fulfillment of the requirement for the degree of **Bachelor of Technology in Mechanical Engineering**, National Institute of Technology, Rourkela, is being carried out under my supervision.

To the best of my knowledge the matter embodied in the thesis has not been submitted to any other university/institute for the award of any degree or diploma.

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ABSTRACT

Time dependent excitation appearing as a variable co-efficient in the governing equation of motion is called parametric excitation. Dynamic stability of elastic systems deal with the study of vibrations, induced by pulsating loads that are parametric with respect to certain forms of deformations. An important aspect in the analysis of parametric dynamic system is the establishment of the regions in the parametric space in which the system becomes unstable; these regions are known as regions of dynamic instability. The boundary separating the stable zones from unstable zones are called stability boundaries and the plot of these boundaries on the parametric space is called the stability diagram. Beams are the simplest structural members in any machine, mechanism or large structure. Layered structures are gaining importance in the present day because of their good damping capacity characteristics. The objective of the proposed research work are to carry out experimental investigations to study the dynamic stability of layered structured beams with bolted joints with different configurations of constraining layers. It is to investigate the following aspects of the dynamic stability of beams.

Experimental has been carried out to validate the theoretical findings. The results are in complete match with the theoretical one.

NOMENCLATURE

Although all the principal symbols used in this thesis are defined in the text as they occur, a list of them is presented below for easy reference. On some occasions, a single symbol is used for different meanings depending on the context and thus uniqueness is lost. The contextual explanation of the symbol at its appropriate place of use is hoped to eliminate the confusion.

English symbols

A	Cross-sectional area of the uniform beam.
b	Width of the beam.
E	Young's modulus of the beam material.
h	Height of the beam.
I	The second moment of inertia.
$[K]$	Global elastic stiffness matrix.
$[K_g]$	Global geometric stiffness matrix.
l	Length of an element.
L	Length of the beam.
$[M]$	Global mass matrix.
$[M]$	Element mass matrix.
$P(t)$	Axial periodic load.
P_s	Static component of the periodic load.
P_t	Time dependent component of the periodic load.
T	Kinetic energy of the beam.
t	Time coordinate.
${}_{(e)}T$	Elemental kinetic energy.
U	Total strain energy of the beam.
${}_{(e)}U$	Elemental potential energy.
v	Transverse displacement of the beam.
w	Transverse displacement of the beam.
x	Axial coordinate.
R	Modal co-ordinate
$[C]$	Damping Matrix

Greek symbols

α	Static load factor.
β	Dynamic load factor.
γ	Shear strain.
ζ	$= x/l$
$\rho_{(2k-1)}$	Mass density of the $(2k-1)$ th constraining elastic layer.

Ω Excitation frequency of the dynamic load component.

Superscripts

(e) Element

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CHAPTER 1

INTRODUCTION

INTRODUCTION

Mechanical joints can have a significant effect on the dynamics of structures that contain them. Bolted or riveted joints cause local stiffness and damping changes and are often the primary source of energy dissipation and damping in assembled structures. In many such structures, damping due to relative interfacial joint motion can account for as much as 90% of the total. Thus, accurate prediction of dynamic response of assembled structures to external excitation often hinges on efficacious modeling of the effect of the joint on structural behavior. Considerable effort has been expended attempting to characterize the non-linear behavior of structures containing joints. Liu and Ewins presented a “more general and more practical” method to extract the “effective” mass, stiffness and damping parameters of a joint element from measured frequency response functions. Note that a common characteristic of all of the above is that no parametric model of the joint is required.

The aim is to develop predictive dynamic parametric models of mechanical joints for reliable structural response analysis. The successful modeling of joints depends on understanding and reproducing the basic physics associated with a jointed interface. Various studies have identified micro- and macro-slip occurring along the interface as the source of change of interface stiffness and energy dissipation, which constitutes the hysteresis mechanism of joints. Typically, the normal interface pressure across a dynamically loaded joint is not uniformly distributed, and micro-slip first occurs in regions where the contact pressure is insufficient to prevent it. The interface is, thus, divided into zones of “stick” and “slip”. As the magnitude of the transmitted load increases, slip zones enlarge and coalesce, resulting in macroslip and the familiar hysteretic force–displacement joint characteristic. This has been demonstrated experimentally for shear lap joints, loaded axially and torsionally, by Gaul and Lenz. The notion that the non-linear hysteresis behavior of a joint is the result of micro- and macro slip occurring along its interface was motivation for developing detailed finite element joint models, which requires solving a contact problem at the interface using an extremely fine mesh.

While it is possible to realistically model the behavior of the joint in this manner, the resulting joint model is impractical for dynamic analysis of an arbitrary structure containing the joint. In fact, it is computationally prohibitive and likely to remain so for some time. Lee et al. proposed a technique to reduce the number of degrees of freedom of a detailed finite element contact model. Chen and Deng noted that finite element analysis can provide a flexible and reliable tool for

understanding and characterizing the non-linear damping behavior of structural joints and proposed to use the finite element method to generate numerical data for a typical slip joint. A lumped-parameter model of small dimension to simulate the non-linear dynamic behavior of a joint and, particularly, its effect upon the surrounding structure has long been deemed desirable. It has been common to represent the friction occurring at contact interfaces by a Coulomb friction model. However, a single Coulomb friction element is only capable of describing either the full slip or full stick situation. Menq et al. developed a one-dimensional, physically motivated micro-slip model that allows partial slip on the friction interface; however, this model is not suited to response analysis of complex structures. At present, non-linear, reduced order, full-joint models that can be effectively used in structural dynamic response analysis do not exist.

CHAPTER 2

LITERATURE REVIEW AND PRESENT WORK

2. LITERATURE REVIEW AND PRESENT WORK:

The environmental interaction with the deformable continuum is usually represented by means of body forces and surface tractions. When the body deforms, dead loads acting on the deformable bodies retain their magnitude as well as their initial direction. In general the forces acting on the body may not always be dead loads. The environmental mechanical action on a body may be due to forces, which are motion and/or time dependent. Such forces are instationary in nature. When these instationary external excitations are parametric with respect to certain form of deformation of the body, they appear as one of the coefficients in the homogeneous governing differential equation of motion of the system. Such systems are said to be parametrically excited and the associated instability of the system is called parametric resonance. Whereas in case of forced vibration of the systems, the equation of motion of the system is inhomogeneous and the disturbing forces appear as in homogeneity. In parametric instability the rate of increase in amplitude is generally exponential and thus potentially dangerous, while in typical resonance due to external excitation the rate of increase in response is linear. More over damping reduces the severity of typical resonance, but may only reduce the rate of increase during parametric resonance. Parametric instability occurs over a region of parameter space and not at discrete points. It may occur due to excitation at frequencies remote from the natural frequencies.

In practice parametric excitation can occur in structural systems subjected to vertical ground motion, aircraft structures subjected to turbulent flow, and in machine components and mechanisms. Other examples are longitudinal excitation of rocket tanks and their liquid propellant by the combustion chambers during powered flight, helicopter blades in forward flight in a free-stream that varies periodically, and spinning satellites in elliptic orbits passing through a periodically varying gravitational field. In industrial machines and mechanisms, their components and instruments are frequently subjected to periodic or random excitation transmitted through elastic coupling elements. A few examples include those associated with electromagnetic and aeronautical instruments, vibratory conveyers, saw blades, belt drives and robot manipulators etc. The system can experience parametric instability, when the excitation frequency or any integer multiple of it is twice the natural frequency that is to say

$$m\omega^2 = \Omega \qquad m = 4, 3, 2, 1, 2\omega$$

The case $\omega = \Omega$ is known to be the most important in application and is called main or principal parametric resonance.

One of the main objectives of the analysis of parametrically excited systems is to establish the regions in the parameter space in which the system becomes unstable. These regions are known as regions of dynamic instability. The boundary separating a stable region from an unstable one is called a stability boundary. Plot of these boundaries on the parameter space is called a stability diagram.

Many machines and structural members can be modeled, as beams with different geometries, like beams of uniform cross-section, tapered beams and twisted beams. These elements may have different boundary conditions depending on their applications. Advances in material science have contributed many alloys and composite materials having high strength to weight ratio. However during manufacturing of these materials, inclusion of flaws affects their structural strength. These flaws can be modeled as localized damage. The modulus of elasticity of the material is greatly affected by the temperature. In high-speed atmospheric flights, nuclear engineering applications, drilling operations and steam and gas turbines, the mechanical and structural parts are subjected to very high temperature. Most of the engineering materials are found to have a linear relationship between the Young's modulus and temperature [44,114]. Geometry of the beam, boundary conditions, localized damage and thermal conditions have greater effect on the dynamic behavior of the beams and hence need to be studied in depth.

CHAPTER 3

THEORETICAL ANALYSIS

3. THEORETICAL ANALYSIS

3.1 Formulation of the problem:

Beams with end conditions such as Fixed-Fixed

The matrix equation for free vibration of axially loaded discretised system is

$$[M]\{\ddot{q}\} + [K_e]\{q\} - P[S]\{q\} = 0 \quad \text{-----}(1)$$

Where, $\{q\}$ = Assemblage nodal displacement vector $[v_i, \theta_i, v_j, \theta_j]^T$.

The dynamic load $P(t)$ is periodic and can be expressed in the form

$$P = P_0 + P_t \cos \Omega t$$

Where Ω is the disturbing frequency, P_0 the static and P_t the amplitude of time dependent component of the load, can be represented as the fraction of the fundamental static buckling load P^* . Hence substituting

$$P = \alpha P^* + \beta P^* \cos \Omega t, \text{ with } \alpha \text{ and } \beta \text{ as static and dynamic load factors respectively.}$$

The equation (1) becomes

$$[M]\{\ddot{q}\} + \left([K_e] - \alpha P^*[S_s] - \beta P^* \cos \Omega t [S_t] \right) \{q\} = 0 \quad \text{-----}(2)$$

Where the matrices $[S_s]$ and $[S_t]$ reflect the influence of P_0 and P_t respectively. If the static and time dependent component of loads are applied in the same manner, then $[S_s] = [S_t] = [S]$

Equation (2) represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type. The development of region of instability arises from Floquet's theory which establishes the existence of the periodic solutions of period T and $2T$,

where $T = \frac{2\pi}{\Omega}$. The boundaries of the primary instability region with period $2T$ are of practical

importance (Bolotin–5) and the solution can be achieved in the form of trigonometric series.

$$q(t) = \sum_{K=1}^{\infty} \left[\{a_k\} \sin \frac{K\theta}{2} t + \{b_k\} \cos \frac{K\theta}{2} t \right] \quad \text{-----}(3)$$

Putting this in equation (2) and if only first term of the series is considered, equating

coefficients of $\sin \frac{\theta}{2} t$ and $\cos \frac{\theta}{2} t$, the equation becomes

$$\left[[K_e] - (\alpha \pm \beta / 2) P^*[S] - \frac{\Omega^2}{4} [M] \right] \{q\} = 0 \quad \text{-----}(4)$$

Equation (4) represents an eigenvalue problem for unknown values of α, β and P^* . This equation gives two sets of eigenvalues (Ω) bounding the regions of instability due to presence of plus and minus sign.

Also this equation (4) represents the solution to a number of related problems.

(i) For free vibration: $\alpha = 0, \beta = 0$ & $\lambda = \frac{\Omega}{2}$

Equation (4) becomes

$$([K_e] - \lambda^2 [M]) \{q\} = 0 \quad \text{-----}(5)$$

(ii) For vibration with static axial load:

$$\beta = 0, \alpha \neq 0, \lambda = \frac{\Omega}{2}$$

Equation (4) becomes

$$([K_e] - \alpha P^*[S] - \lambda^2 [M]) \{q\} = 0 \quad \text{-----}(6)$$

- (iii) For static stability: $\alpha = 1, \beta = 0, \lambda = \frac{\Omega}{2}$

Equation 4 becomes

$$([K_e] - P^*[S])\{q\} = 0 \quad \text{-----}(7)$$

- (iv) For dynamic stability, when all terms are present

$$\text{Let } \Omega = \left(\frac{\Omega}{\omega_1} \right) \quad \omega_1$$

Where ω_1 is the fundamental natural frequency as obtained from the solution of equation

(5). Equation (4) then becomes

$$\left([K_e] - \left(\alpha \pm \frac{\beta}{2} \right) P^*[S] \right) \{q\} = \theta \frac{\omega_1^2}{4} [M] \{q\} \quad \text{-----}(8)$$

$$\text{Where, } \theta = \left(\frac{\Omega}{\omega_1} \right)^2$$

The fundamental natural frequency ω_1 and critical static buckling load P^* can be solved using the equations (5) and (7) respectively. The regions of dynamic instability can be determined from equation (8).

3.2 FEM Analysis

The input file, *stiffmat1.m*, *bolts.m*, and *boundary.m* are executed in succession within Matlab to form the model mass and stiffness matrices. These matrices can then be used to describe an undamped eigenproblem [16,23] that will give the free vibration characteristics of the structure:

$$([K] - \omega^2 [M])[x] = 0 \quad (1)$$

where $[K]$ is the stiffness matrix, $[M]$ is the mass matrix, and ω^2 is an eigenvalue of the problem. The eigenvalues give the undamped natural frequencies of the structure, while the eigenvectors describe the mode shapes. For lightly damped structures, the undamped natural

frequencies are very close to the actual (damped) values.

FEMs can also be used to generate analytical FRFs. This can be done a number of ways. One way is to use the model to generate time responses to a given excitation. The time responses can then be used to generate FRFs. The process is as follows:

The mass and stiffness matrices described by the FEM represent a system of differential equation of the form:

$$[M]\{\ddot{x}\} + [K]\{x\} = [F] \quad (2)$$

where $\{\ddot{x}_o\}$ is a vector of nodal accelerations, $\{x\}$ is a vector of nodal displacements, and $\{F\}$ is

a vector of forces applied at each node. This problem is accompanied by a set of initial positions $\{x\}$ and initial velocities $\{v_o\}$. In general, the system will be coupled, which makes a solution difficult. The modal analysis procedure is a coordinate transformation where all physical coordinates $\{x\}$ are transformed to modal coordinates $\{r\}$. This results in a system similar to (2):

$$\{\ddot{r}\} + [\Lambda]\{r\} = \{f\} \quad (3)$$

where $\{r\}$ is the transformed version of $\{x\}$ and $[\Lambda]$ is a diagonal matrix containing the values ω^2

The transformation to modal coordinates is accomplished through transformation matrices defined by $[K]$ and $[M]$. First, the matrix $[M]^{-1/2}$ is calculated. This matrix is then used to transform the stiffness matrix:

$$[\tilde{K}] = [M]^{-1/2}[K][M]^{-1/2} \quad (4)$$

An eigensolution is then performed on K and the eigenvectors are arranged in order as columns in a new matrix $[P]$:

$$[P] = [v_1 v_2 v_3 \dots v_n] \quad (5)$$

The transformation matrices can then be calculated as

$$[S] = [M]^{-1/2}[P]$$

$$[S]^{-1} = [P]^T [M]^{-1} \quad (7)$$

The transformation to modal coordinates can then be accomplished as

$$\{r\} = [S]^{-1} \{x\} \quad (8)$$

The transformation described by (8) gives the system in (3), in which the differential equations are not coupled. This means that they can each be solved independently with a solution of the form

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The transformation described by (8) gives the system in (3), in which the differential equations are not coupled. This means that they can each be solved independently with a solution of the form

16

$$r_n(t) = A_n \sin(\omega_n t + \phi_n) \quad (9)$$

which is an undamped oscillatory motion (this solution assumes that all elements in $\{f\}$ are zero, giving homogeneous equations). The constants A_n and ϕ_n depend on the initial conditions of the problem (each vector of initial conditions must also be transformed into modal coordinates).

Once the solution vector $\{r\}$ is known, it can be transformed back to physical coordinates with the transformation

$$\{x\} = [S] \{r\} \quad (10)$$

This solution vector $\{x\}$ gives the time responses of each DOF due to the initial conditions. Note that as each time response is an undamped motion, it will continue to oscillate and never decay to zero. This is obviously unrealistic. Some form of damping must be incorporated into the equations to give a more realistic description:

$$[M] \{\ddot{x}\} + [C] \{\dot{x}\} + [K] \{x\} = \{F\} \quad (11)$$

where $[C]$ is the damping matrix.

The finite element formulation gives the mass and stiffness matrices, but the damping matrix must be described in another way. In the modal analysis process outlined above, the damping matrix will often not decouple when the transformation is applied, preventing a solution.

Proportional damping is a type of damping where the damping matrix is a linear combination of

the mass and stiffness matrices:

$$[C] = \alpha [M] + \beta [K] \quad (12)$$

where α and β are constants. This type of damping is relatively easy to compute and lends itself to an easy solution by modal analysis. When the damping matrix fits this formula, equation (11) will decouple into the form

$$\{\ddot{r}\} + 2[\zeta_n][\omega_n]\{\dot{r}\} + [\lambda]\{r\} = \{f\} \quad (13)$$

where $[\zeta_n]$ is a diagonal matrix containing the modal damping values and $[\omega_n]$ is a diagonal matrix containing the values ω_n . Each equation in this system will have a solution of the general form

$$r_n(t) = A_n e^{-\zeta_n \omega_n t} \sin(\omega_{dn} t + \varphi)$$

where again A_n and φ_n are constants that depend on the initial conditions (this solution is only true for the unforced system). This solution form is an exponentially decaying oscillatory motion, which could be a close approximation for some structures.

The constants α and β in equation (12) can be chosen to provide specific damping values, or calculated from known damping values according to the equation

$$\zeta_n = \frac{\alpha}{2\omega_n} + \frac{\beta\omega_n}{2} \quad (15)$$

CHAPTER 4

EXPERIMENTAL WORK

4.0 EXPERIMENTAL WORK

4.1 Introduction

The aim of the experimental work is to establish experimentally the stability diagrams for multi-layered structural beams with bolted joints under axial loading. For multi-layered beams, the stability diagrams have been experimentally established for 2, 3 layered beams. The theoretical and experimental stability diagrams have been compared to assess the accuracy of the theoretical results.

4.2 Description of the experimental set up

The main components are as follows:

1. Frame.
2. Adjustable Beam.
3. Electrodynamic shaker (EDS).
4. Load Cell.
5. Vibration Pick-ups.
6. Power Amplifier Panel.
7. Screw Jack.
8. Signal Analyzer (computer).

The schematic diagram of the equipments used for the experiment and photographic view of the experimental set up is shown in the fig. The set up consists of a framework fabricated from steel channel sections by welding. The frame is fixed in vertical position to the foundation by means of foundation bolts and it has the provision to accommodate beams of different lengths. The periodic axial load $P_i \cos \Omega t$ is applied to the specimen by a 500N capacity electrodynamic shaker (Saraswati Dynamics, India, Model no. SEV-005). The static load can be applied to the specimen by means of a screw jack fixed to the frame at the upper end. The applied load on the specimen is measured by a piezoelectric load cell (Brüel & Kjaer, model no. 2310-100), which is fixed between the shaker and the specimen. The vibration response of the test specimen is measured by means of vibration pick-ups (B&K type, model no. MM-0002). The signals from the pickups and load cell are observed on a computer through a six-channel data acquisition system (B&K, 3560-C), which works on Pulse software platform (B&K 7770, Version 9.0).

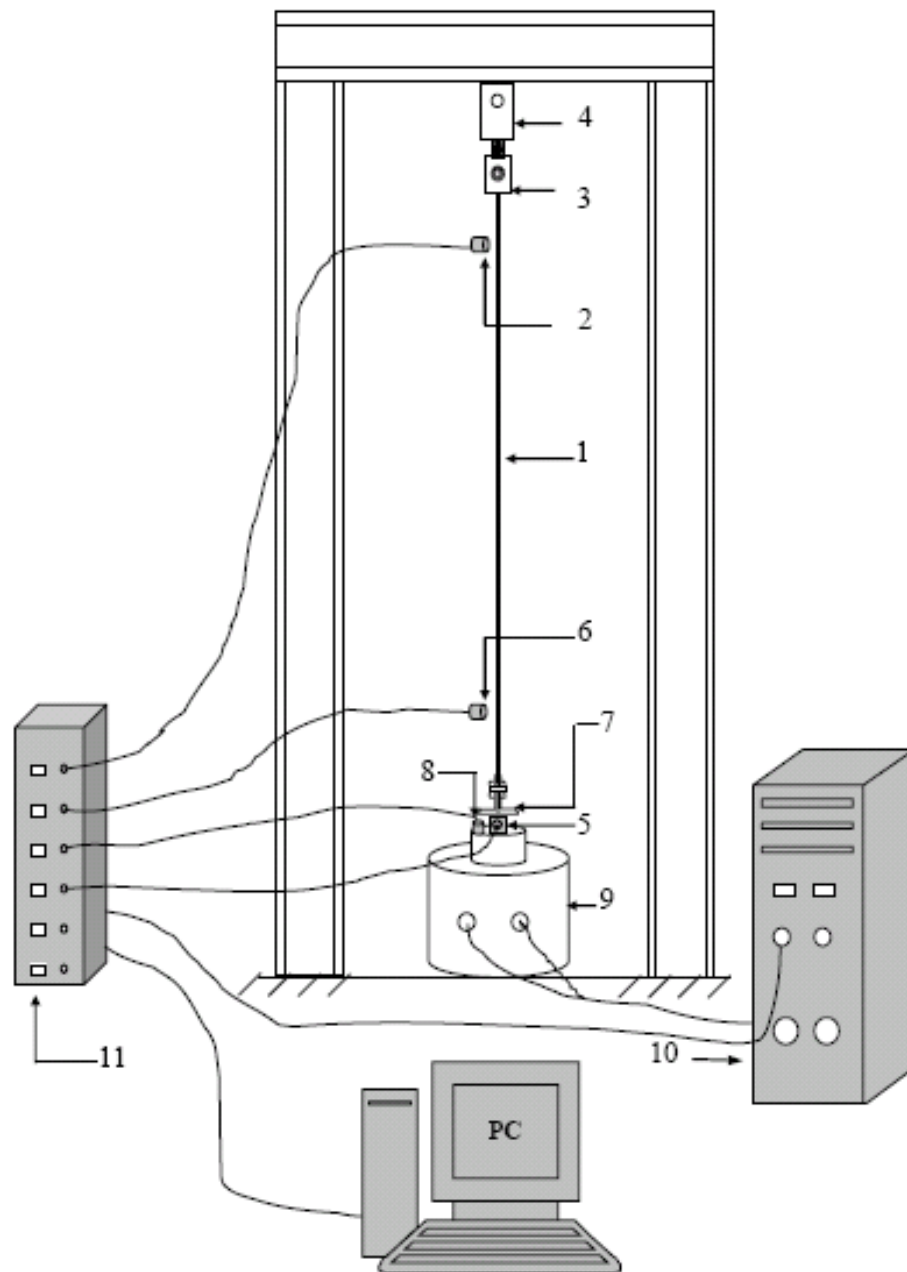


Fig 4.1 Schematic diagram of the test set up:1. Specimen, 2. Upper Pick-up, 3. Upper support, 4. Screw Jack, 5. Load Cell, 6. Lower Pickup, 7. Lower support, 8. Accelerometer, 9. Vibration generator, 10. oscillator and amplifier, 11. Data acquisition system.



Fig 4.2 *Photograph of the experimental setup.*



Fig 4.3 *Photographs of attachments for clamped end*

4.3 PREPARATION OF SPECIMEN:

For the Experimental validation of the dynamic stability of layered beams, multi-layered specimens were made up of mild steel (M.S) and galvanized iron (G.I). The general preparation procedure for the two types of materials was the same except for change in the layer thickness. The length of a single strip of beam was calculated from the EULER'S Equation for Buckling load:

$$L = ((4\pi^2 EI)/\rho)^{1/2}$$

Multi-layered beams were prepared by joining no. of strips with the help of nut, bolt and washer assembly. The bolts were placed about 5cm apart and the torque on each bolt was assured to be uniform with the help of a torque meter.

The length of each layer was kept uniform. For a beam of thickness t mm, corresponding beam was prepared having n no. of layers with thickness t_1 mm.

$$\text{where } t = n \cdot t_1 \text{ (in mm)}$$

Table 4.1 M.S, $t = 0.98$ mm(t)

t_1 (mm)	No. of pieces	Corresponding Thickness
0.16	6	0.96
0.32	3	0.96
0.5	2	1

Table 4.2 M.S, $t = 1.26$ mm(t)

t_1 (mm)	No. of pieces	Corresponding Thickness
0.16	8	1.28
0.32	4	1.28
0.63	2	1.26

Table 4.3 G.I, $t = 1.16$ mm(t)

t_1 (mm)	No. of pieces	Corresponding Thickness
0.16	7	1.12
0.32	3	0.9
0.5	2	1.0



Fig4.4: Photograph showing the prepared specimen

4.4 Experimental procedure

An oscillator cum power amplifier unit drives the electrodynamic vibration shaker used for providing the dynamic loading. The beam response was recorded by the non-contacting vibration pickups. For straight beams with fixed-fixed end conditions two pickups, one at each end of the beam were used.

Initially the beam was excited at certain frequency and the amplitude of excitation was increased till the response was observed. Then the amplitude of excitation was kept constant and frequency of excitation was changed in steps of 0.1 Hz. The excitation frequency was controlled with the help of Pulse software. The generator module of the

Pulse software can produce an output signal of specified frequency; this signal is fed to the shaker through its control unit. The experimental boundaries of instability regions were marked by the parameters (P_i, Ω) , which were measured just before a sudden increase of the amplitude of the lateral vibration. The order of increase in amplitude of lateral vibration has been taken to be around 4.0, to record the boundary frequency of instability. For accurate measurement of the excitation frequency an accelerometer was fixed to the moving platform of the exciter, its response was observed on computer in the frequency domain. The dynamic load component of the applied load was measured from the response curve of the load cell. For ordinary beams the excitation frequency was divided by the reference frequency ω_1 to get the non-dimensional excitation frequency (Ω/ω_1) .

CHAPTER 5

RESULTS AND CONCLUSIONS

RESULTS:

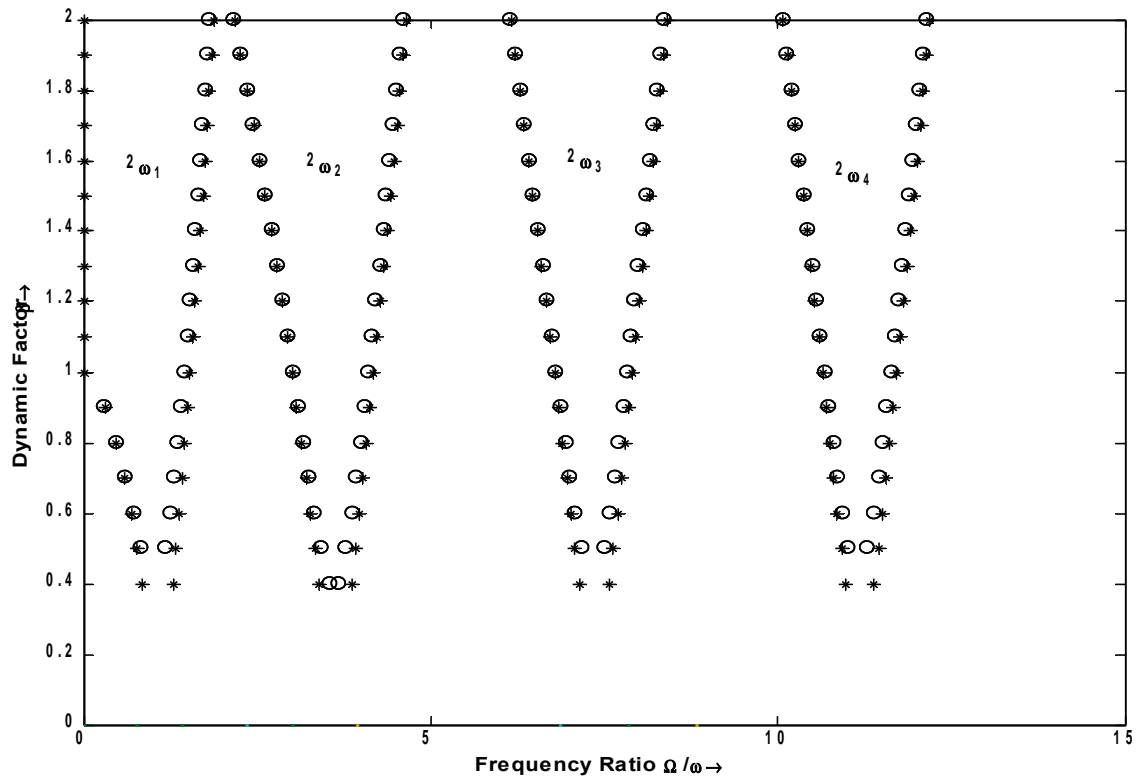
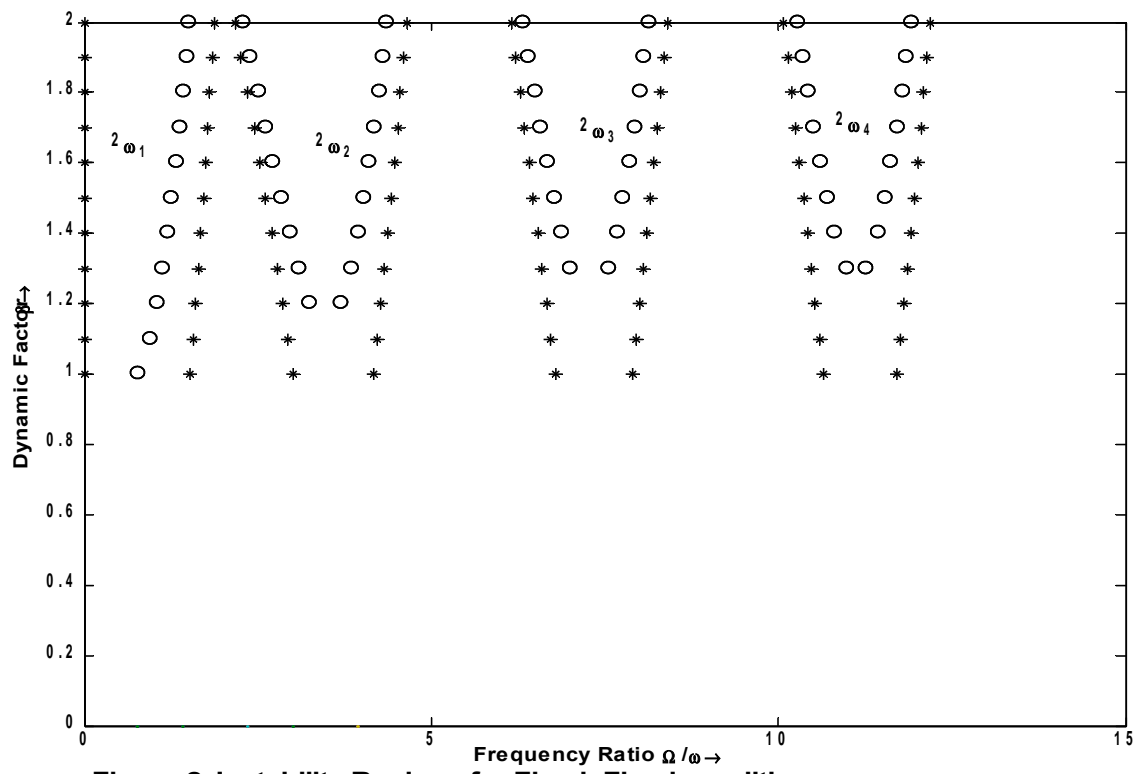


Figure-1 Instability regions for Fixed -Fixed Condition ,
'o' With Damping , '*' Without Damping



**Figure-2 ,Instability Regions,for Fixed -Fixed condition,
,*,without damping,'o' with damping $\xi_1=0.3, \xi_2=0.3$**

CONCLUSIONS:

From the obtained results it is seen that with increase in damping the area of instability regions goes on decreasing. Moreover it is inferred that for a higher value of damping instability regions start occurring after a comparatively higher value of applied load. The damping effect produced varies increasingly with the no. of layers of beams used that is more the number of layers of beam more is the damping effect. In the above graphs $\xi = 0.1$ was obtained from a two layered beam whereas a damping value of $\xi = 0.3$ was obtained in case of a four layered beam. By using multilayered beams the damping increases and instability regions are produced after a comparatively higher value of dynamic loading as is evident from the graphs. because of increased damping and hence being less prone to vibrational damages multilayered beams can be trusted for practical applications like in aerospace engineering against the usage of single layered beams.

CHAPTER 6

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